Rock Damage and Healing Mechanics

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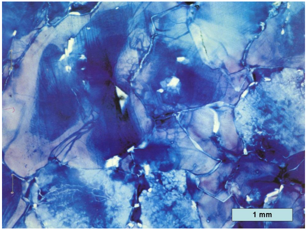


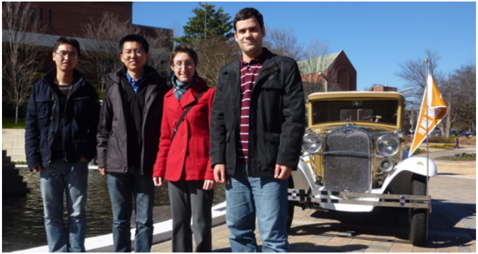
Schleder et al., 2013

Wikipedia, 08/2013

Wordpress, 08/2013

Geophysics Department Seminar, Stanford University, May 15, 2014

Acknowledgements



Damage Poro-Mechanics Laboratory Team (DeeP MeLT)
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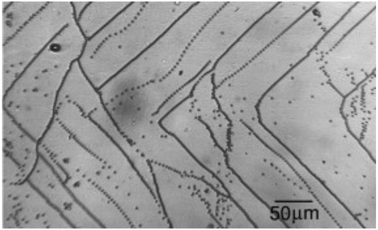
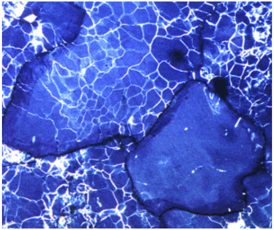
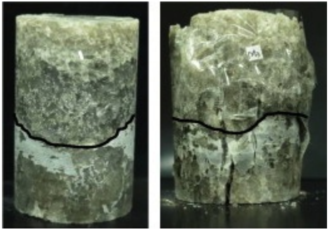
Scales of discontinuities in salt rock



10^{-5}m Barber et al., 2010



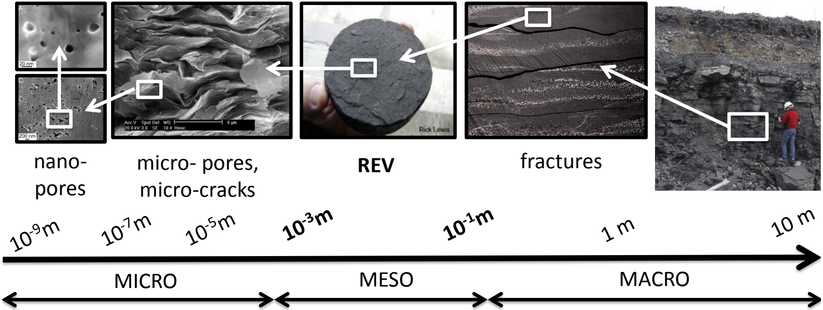
10^{-3}m Schleder et al., 2013



10^{-1}m Xie et al., 2011

10^{+1}m Wordpress, 08/2013

Multi-scale crack propagation in shale



- Micro-scale
- Linear Elastic Fracture Mechanics (LEFM)
- Discrete Element Method (DEM)

- Meso-scale
- Continuum Damage Mechanics (CDM)
- Finite Element Method (FEM)

- Macro-scale
- Linear Elastic Fracture Mechanics (LEFM)
- Extended Finite Element Method (XFEM)

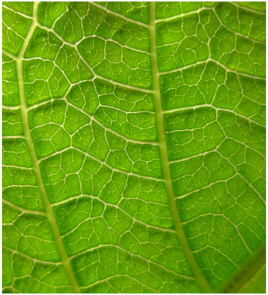
To Scale, or Not To Scale...

NATURE NEWS BLOG

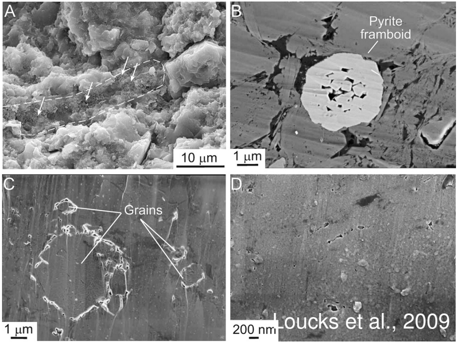
Mesoscale physics buzz eludes exact definition

01 Mar 2012 | 20:18 GMT | Posted by Eugenie Samuel Reich | Category: Physics & Mathematics, Policy

The future of physics will depend crucially on researchers' ability to tackle phenomena at the mesoscale, an enigmatic realm that bridges quantum and classical physics. On that point, speakers at a special session and Town Hall meeting yesterday at the American Physical Society's March meeting in Boston, Massachusetts, were in agreement. But what exactly is the mesoscale, and what kind of research is needed to understand it? That question brought forth multiple overlapping answers. "This is a buzz word, so you're free to define it any way you want," said physics Nobel Laureate Robert Laughlin of Stanford University in California, whose own answer was that the mesoscale was the scale of life, as described by emergent laws of nature that have to be discovered, rather than deduced.



The Mesoscope

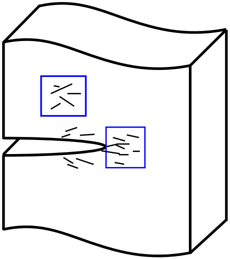


- Mesoscale : “where systems are on the boundary between interacting and isolated systems”
- impact of “morphology of surfaces”

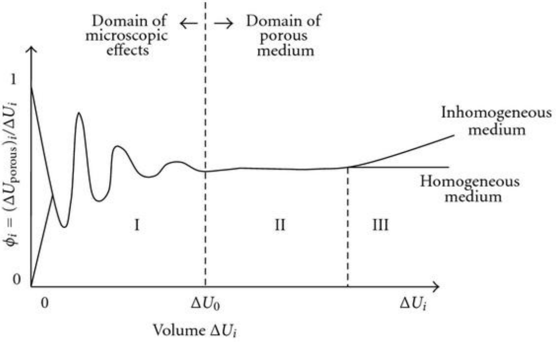
“Areas of study that could benefit from being considered mesoscale physics [...] include the evolution of defects and accumulation of damage in materials [..], the transport of liquids through mesoporous media such as rock - that might benefit efforts at carbon dioxide sequestration or fracking [...].

Defining damage starts by defining the REV

Continuum Damage Mechanics (CDM) is used at mesoscopic scale of the Representative Elementary Volume (REV)



Representative Elementary Volume (REV)



[Borges et al., 2012]

The Two First Laws of Thermodynamics

The first law of thermodynamics is an energy balance equation. It states that the change in the internal energy (U) of a closed system is equal to the energy brought by external actions to the system (P) minus the amount of heat that the system gives to the environment (Q).

$$\dot{P} - \dot{Q} = \dot{U}$$

Entropy provides a measure of the amount of energy that cannot be used to do work. The time-dependent entropy change (S) can be divided into external entropy (S_e) and internal entropy (S_i).

$$\dot{S} = \dot{S}_e + \dot{S}_i = -\frac{\dot{Q}}{\tau} + \dot{S}_i$$

The second law of thermodynamics states that the energy that cannot be used to work is dissipated because of irreversible microstructure changes :

$$\dot{S}_i \geq 0 \quad \Rightarrow \quad \dot{S} \geq -\frac{\dot{Q}}{\tau}$$

Inequality of Clausius-Duhem

Locally, the rate of heat brought to the system from the surroundings is related to entropy by :

$$\frac{dS}{dt} \geq -\nabla_X \cdot (\frac{q}{\tau})$$

We introduce Helmholyz free energy :

$$\Psi = U - \tau S$$

Combining the two first laws of thermodynamics :

$$\dot{P} - \dot{\tau} S - \dot{\Psi} - \frac{q}{\tau} \cdot \nabla \tau \geq 0$$

Using the Principle of Virtual Work, we get the so-called Inequality of Clausius-Duhem :

$$\sigma : \dot{\epsilon} - \dot{\tau} S - \dot{\Psi} - \frac{q}{\tau} \cdot \nabla \tau \geq 0$$

Thermodynamic Conjugation Relationships

For reversible mechanical processes, in the absence of heat transfer :

$$\sigma : \dot{\epsilon}^e - \dot{\tau} S_e \cdot \nabla\tau - \dot{\Psi} = 0$$

Helmholtz free energy is sought in the form : $\Psi = \Psi(\epsilon^e, \tau)$:

$$\left(\boldsymbol{\sigma} - \frac{\partial\Psi}{\partial\boldsymbol{\epsilon}^e}\right)\dot{\epsilon}^e - \left(S_e + \frac{\partial\Psi}{\partial\tau}\right)\dot{\tau} = 0$$

And the thermodynamic conjugation relationships write :

$$\boldsymbol{\sigma} = \frac{\partial\Psi}{\partial\boldsymbol{\epsilon}^e}, \qquad S_e = -\frac{\partial\Psi}{\partial\tau}$$

For irreversible processes, the reduced Inequality of Clausius-Duhem writes :

$$\Phi_M + \Phi_T \geq 0, \quad \Phi_M = \sigma : (\dot{\epsilon} - \dot{\epsilon}^e) + \boldsymbol{\chi} : \dot{\xi} \geq 0, \quad \Phi_T = -\frac{q}{\tau} \cdot \nabla\tau \geq 0$$

$\dot{\epsilon} - \dot{\epsilon}^e$: irreversible deformation (plastic, viscoplastic, residual crack opening...)
$\boldsymbol{\chi}$, $\boldsymbol{\xi}$: internal variables (damage, hardening variables such as dilatancy...

Continuum Damage Mechanics

Problem : model the influence of crack debonding (cohesive damage) and crack opening (adhesive damage) on deformation and stiffness

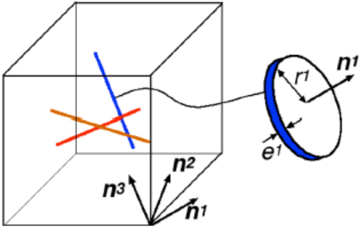
Solution in Continuum Damage Mechanics : stiffness softening + irreversible crack deformation [Halm & Dragon, 1998, Abu Al Rub & Voyiadjis, 2003]

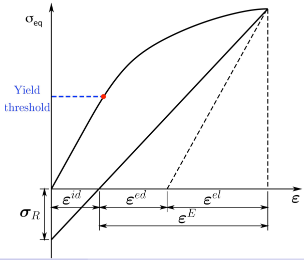
Damage variable (REV scale) : second-order crack-density tensor [Kachanov, 1992] :

$$\mathbf{D} = \sum_{k=1}^{3} D_k \mathbf{n}^k \otimes \mathbf{n}^k$$

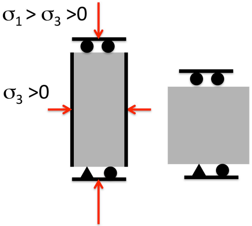
Stress and strain decomposition :

$$\boldsymbol{\sigma} = \mathbf{C}(\mathbf{D}) : \boldsymbol{\epsilon}^E + \boldsymbol{\sigma}_R, \quad \boldsymbol{\epsilon}^E = \boldsymbol{\epsilon}^{el} + \boldsymbol{\epsilon}^{ed} = \boldsymbol{\epsilon} - \boldsymbol{\epsilon}^{id}$$

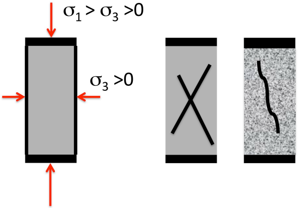




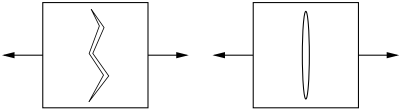
Damage due to tensile and compressional micro-cracks



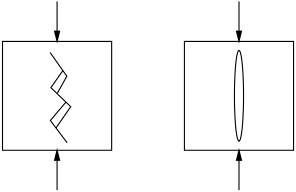
(a) Non Frictional Boundaries



(b) Frictional Boundaries



Splitting effects [Ortiz, 1985]



Crossing effects [Ortiz, 1985]

Deviatoric Stress Induced Damage (DSID) Model

Model Formulation

Hyper-elasticity

1. Free energy

$$\epsilon^E = \frac{\partial G_s(\boldsymbol{\sigma}, \boldsymbol{D})}{\partial \boldsymbol{\sigma}}$$

2. $f_d(\boldsymbol{D}, \boldsymbol{Y})$

$$\epsilon = \epsilon^E + \epsilon^{id}$$

3. $g_d(\boldsymbol{D}, \boldsymbol{Y})$

4. Flow rule

1. Free energy [4]

$$G_s = \frac{1}{2}\boldsymbol{\sigma}:\mathbb{S}_0:\boldsymbol{\sigma} + a_1 Tr\boldsymbol{D}(Tr\boldsymbol{\sigma})^2 + a_2 Tr(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}\cdot\boldsymbol{D}) + a_3 Tr\boldsymbol{\sigma} Tr(\boldsymbol{D}\cdot\boldsymbol{\sigma}) + a_4 Tr\boldsymbol{D} Tr(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma})$$

2. Damage function

$$f_d = \sqrt{J^*} - \alpha I^* - k, \quad I^* = (\mathbb{P}_1:\boldsymbol{Y}):\boldsymbol{\delta},$$

$$J^* = \frac{1}{2}\left(\mathbb{P}_1:\boldsymbol{Y} - \frac{1}{3}I^*\boldsymbol{\delta}\right):\left(\mathbb{P}_1:\boldsymbol{Y} - \frac{1}{3}I^*\boldsymbol{\delta}\right),$$

$$\mathbb{P}_1(\boldsymbol{\sigma}) = \sum_{p=1}^{3}\left[H\left(\sigma^{(p)}\right) - H\left(-\sigma^{(p)}\right)\right]\boldsymbol{n}^{(p)}\otimes\boldsymbol{n}^{(p)}\otimes\boldsymbol{n}^{(p)}\otimes\boldsymbol{n}^{(p)}$$

3. Damage potential

$$g_d = \sqrt{\frac{1}{2}(\mathbb{P}_2:\boldsymbol{Y}):(\mathbb{P}_2:\boldsymbol{Y})},$$

$$\mathbb{P}_2(\boldsymbol{\sigma}) = \sum_{p=1}^{3} H\left[\max_{q=1,2,3}\left(\sigma^{(q)}\right) - \sigma^{(p)}\right]\boldsymbol{n}^{(p)}\otimes\boldsymbol{n}^{(p)}\otimes\boldsymbol{n}^{(p)}\otimes\boldsymbol{n}^{(p)}$$

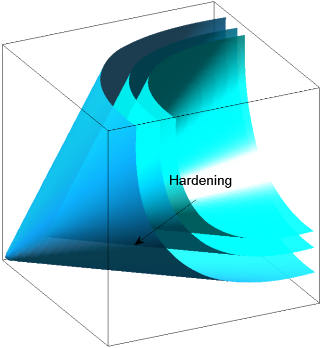
4. Flow rule

$$\dot{\boldsymbol{\epsilon}}^{id} = \dot{\lambda}_d\frac{\partial g_d}{\partial\boldsymbol{\sigma}} = \dot{\lambda}_d\frac{\partial g_d}{\partial\boldsymbol{Y}}:\frac{\partial\boldsymbol{Y}}{\partial\boldsymbol{\sigma}}$$

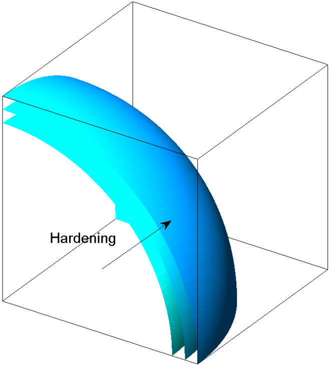
$$\dot{\boldsymbol{D}} = \dot{\lambda}_d\frac{\partial f_d}{\partial\boldsymbol{Y}}$$

[Xu & Arson, *Int. J. Comput. Meth.*, 2014]

Thermodynamic potentials in Generalized Stress Space



Damage function in $\mathbb{P}_1$: **Y** space.

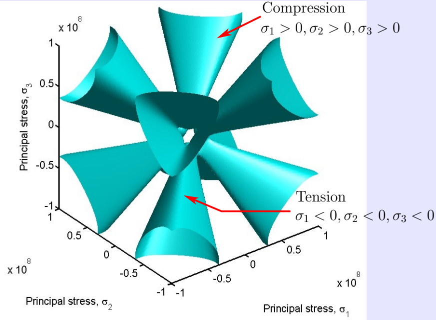


Damage potential in $\mathbb{P}_2$: **Y** space.

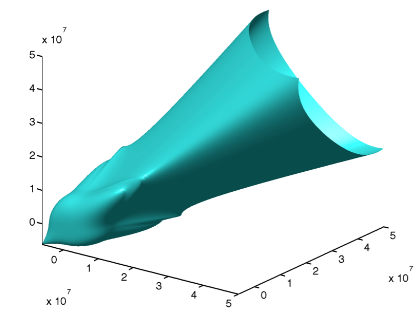
[Arson, *Mech. Res. Comm.*, under review]

DSID Damage Function

New Damage Surface



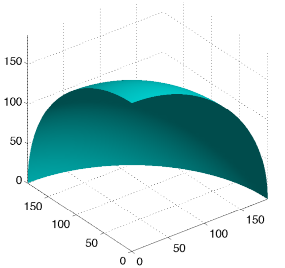
Damage function in **Y** space



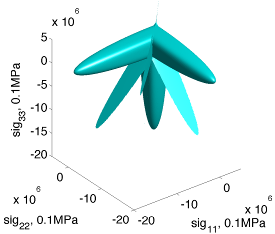
Damage function in stress space $\boldsymbol{\sigma}$

Convexity of Potentials in Generalized Stress Space

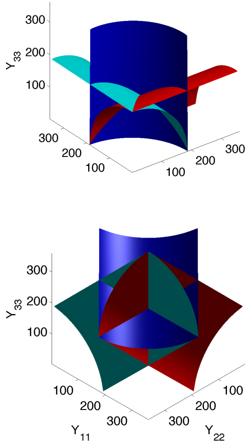
In $\mathbb{P}_2$: **Y** space :



In σ space :



In **Y** space :



$$g_d = \sqrt{\frac{1}{2}(\mathbb{P}_2 : \mathbf{Y}) : (\mathbb{P}_2 : \mathbf{Y})} - C_2$$

The projection tensor $\mathbb{P}_2$ is introduced to represent both "crossing" and "splitting" effects :

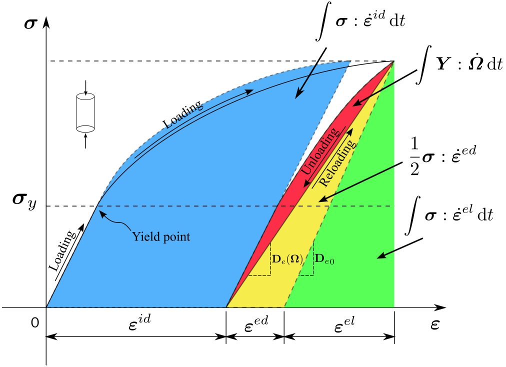
$$\mathbb{P}_2 = \sum_{p=1}^{3} H\left[\max_{q=1}^{3}(\sigma^{(q)}) - \sigma^{(p)}\right] \mathbf{n}^{(p)} \otimes \mathbf{n}^{(p)} \otimes \mathbf{n}^{(p)} \otimes \mathbf{n}^{(p)}$$

Forms of Energy dissipation

The propagation of damage results in :

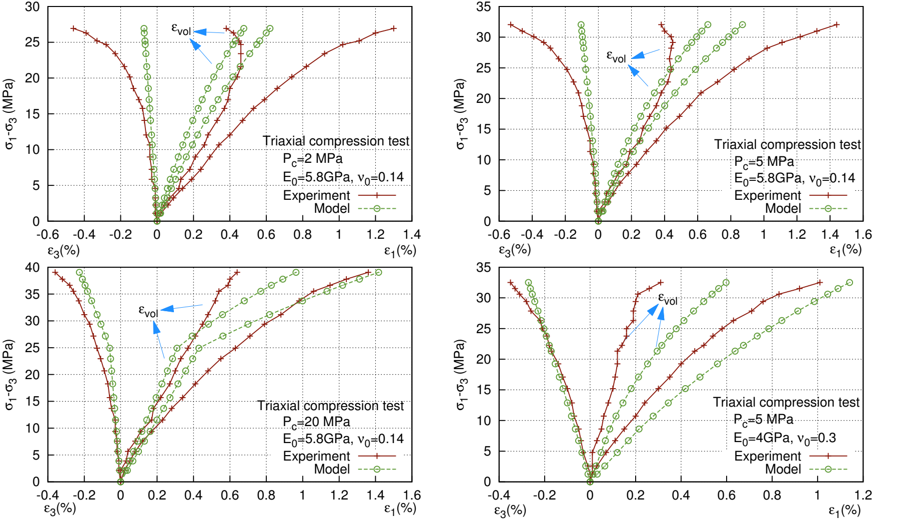
- crack debonding and opening without residual deformation
- crack opening producing inelastic deformation

$$\int \boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}}\, dt - \frac{1}{2}\boldsymbol{\sigma} : \boldsymbol{\epsilon}^E = \int \dot{\Phi}_d\, dt = \int \boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}}^{id}\, \mathrm{d}t + \int \boldsymbol{Y} : \dot{\boldsymbol{\Omega}}\, dt \geq 0$$



[Xu & Arson, *Comput. & Geotech.*, in preparation]

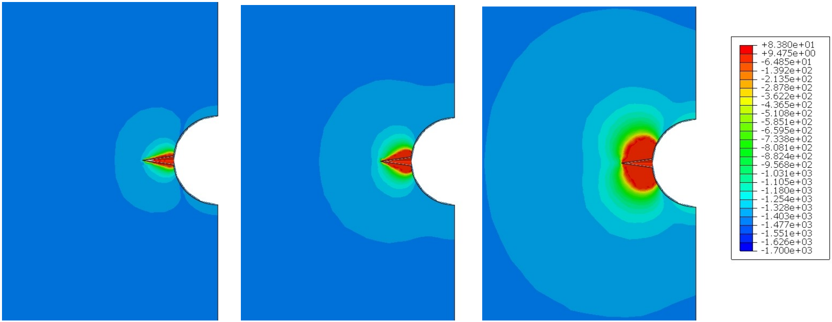
Stress-strain curves after Calibration - Eastern France Clay Rock



[Bakhtiary, Xu & Arson, *Int. J. Rock Mech. Min. Sci.*, 2014]

Problem 1 : Borehole Pressurization

Evolution of the damage zone upon pressurization - Cross-sectional view

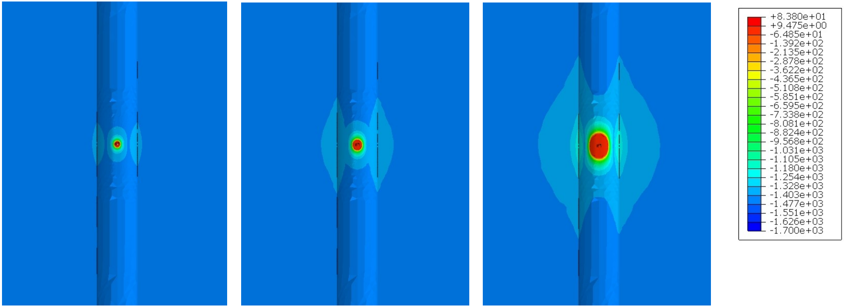


$p = 20$ MPa $\quad$ $p = 50$ MPa $\quad$ $p = 200$ MPa

[Xu, Arson & Busetti, *ASCE GeoCongress*, 2014]

Problem 1 : Borehole Pressurization

Evolution of the damage zone upon pressurization - Longitudinal view

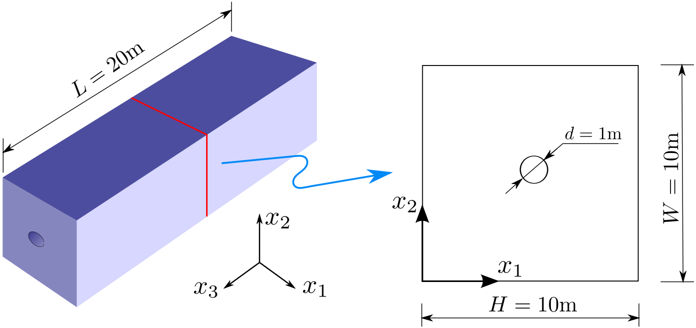


$p = 20$ MPa $\quad$ $p = 50$ MPa $\quad$ $p = 200$ MPa

[Xu, Arson & Busetti, *ASCE GeoCongress*, 2014]

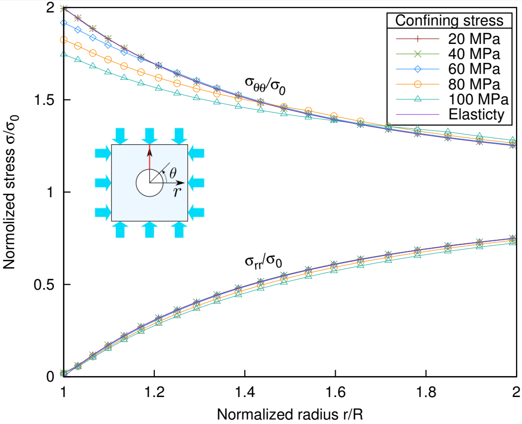
Problem 2 : Excavation of a Circular Cavity

Finite Element simulations are performed in three dimensions, and analyses are restricted to a central cross-section of the domain (right), which can be considered in a state of plane strain.



[Xu & Arson, *Rock Mech. & Rock Eng.*, under review]

Comparison with Analytic Elastic Solution



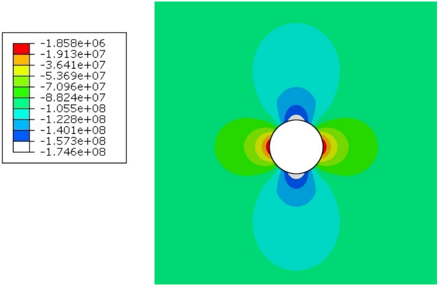
For $r = 0.5 \sim 1$ m ; $\theta = \frac{\pi}{2}$; $R = 0.5$.

According to the theory of elasticity :

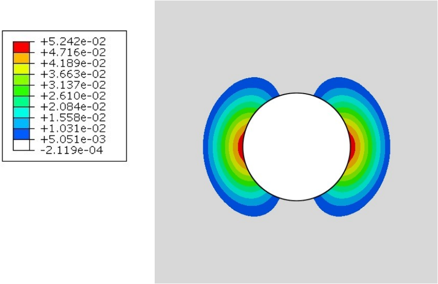
$$\sigma_{\theta\theta} = \sigma_0 \left[1 + \left(\frac{R}{r}\right)^2\right] - \left(\frac{R}{r}\right)^2 \sigma_p, \quad \sigma_{rr} = \sigma_0 \left[1 - \left(\frac{R}{r}\right)^2\right] + \left(\frac{R}{r}\right)^2 \sigma_p$$

Anisotropy of Stress and Damage Induced by Excavation

Stress and damage distribution after excavation in a rock mass subjected to a confining pressure of $\sigma_0 = 100$ MPa in the far field.



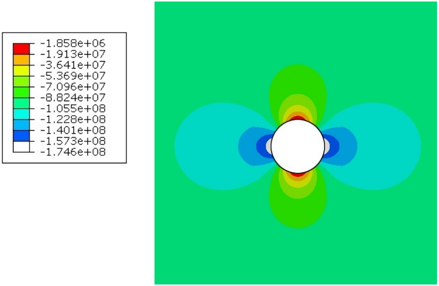
(c) Horizontal stress (σ_{11}, Pa)



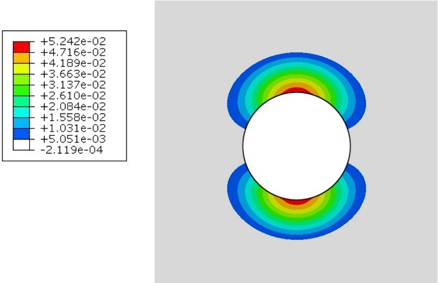
(d) Vertical cracks (Ω_{11})

Anisotropy of Stress and Damage Induced by Excavation

Stress and damage distribution after excavation in a rock mass subjected to a confining pressure of $\sigma_0 = 100$ MPa in the far field.



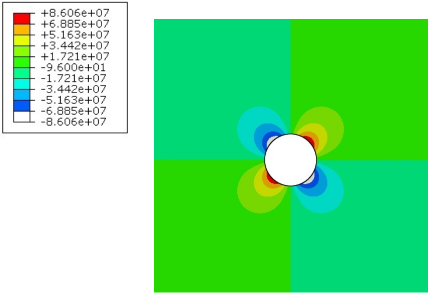
(e) Vertical stress (σ_{22}, Pa)



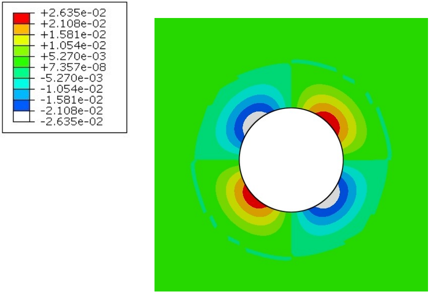
(f) Horizontal cracks (Ω_{22})

Anisotropy of Stress and Damage Induced by Excavation

Stress and damage distribution after excavation in a rock mass subjected to a confining pressure of $\sigma_0 = 100$ MPa in the far field.



(g) Shear stress (σ_{12}, Pa)

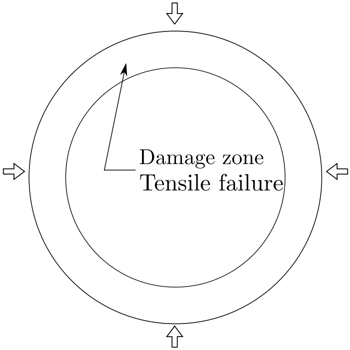


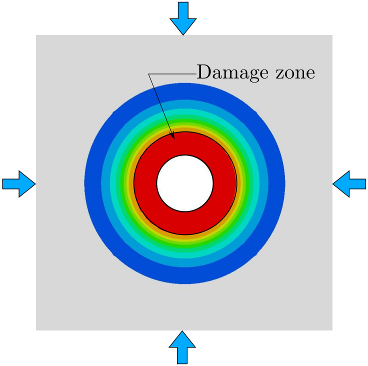
(h) Shear damage (Ω_{12})

Application : Stress Back-Analysis from the EDZ Model

Isotropic stress conditions [Wang, 2014]

- Jointed rock subjected to isotropic confinement
- Cracks parallel to the cavity wall

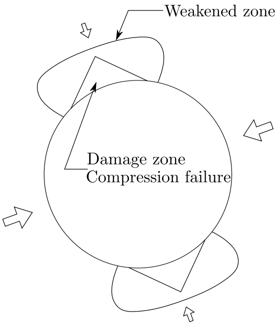


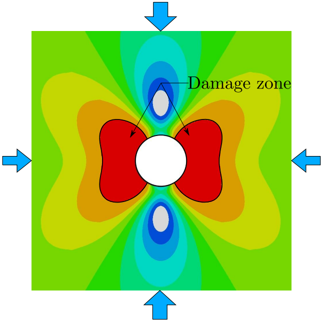


Application : Stress Back-Analysis from the EDZ Model

Anisotropic stress conditions [Martin *et al.*, 1997 ; Shao *et al.*, 1999 ; Martino *et al.*, 2004 ; Read, 2004 ; Everitt and Lajtai, 2004 ; Cai and Kaiser, 2014]

- Crack propagation influenced by heterogeneous rock texture (granite)
- Tensile cracks are generated in planes parallel to the direction of the maximum principal stress (minimum compression) ; the failure zone spreads along the direction of the minimum principal stress, apart from compressive cracks





Microscopic Processes versus Macroscopic Dilatancy Boundary

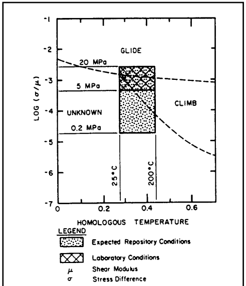
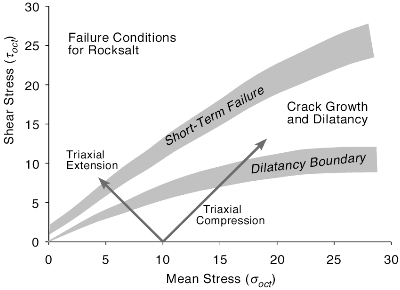


Fig. 17. Deformation mechanism map for WIPP salt [19].

[Senseny et al., 1992]



[Schulze, 2007]

- isochoric dislocation processes
  $\rightarrow$ isochoric viscoplastic deformation
- dilatant micro-cracking
  $\rightarrow$ crack-induced dilatant volumetric deformation
- fluid-assisted Diffusive Mass Transfer
  $\rightarrow$ contractant “healing” deformation

Concept of Dilatancy Boundary
[Hunsche & Hampel, 1999 ; Hou, 2003]

- dilatant crack-induced deformation above the boundary
- contractant “healing” deformation below the boundary
- no volume change within the dilatancy boundary zone

"Deformation Healing" versus "Stiffness Recovery"

	Miao et al., 1995	**Chan et al., 1998**	**Hunsche & Hampel, 1999**	**Hou, 2003**	**Zhou et al., 2011**
Damage Inelastic Deformation	none	stress-induced anisotropic	isotropic dilatancy	stress-induced anisotropic	none (plasticity)
Healing Inelastic Deformation	none	stress-induced anisotropic	isotropic contractancy	stress-induced anisotropic	none
Damaged Elastic Properties	damaged stiffness tensor = internal variable	isotropic softening	none	isotropic softening	isotropic softening
Healed Elastic Properties	isotropic hardening	isotropic hardening	none	isotropic hardening	none

Modeling Objectives :

- definition of internal variables to model crack debonding, opening, closure and rebonding
- prediction of damage- and healing- induced anisotropy of stiffness and deformation
- determination of rock textures favorable to healing

Thermo-mechanical Damage Model

Thermo-mechanical free energy
(modified from [Halm & Dragon, 1998])

$$\Psi_S(\varepsilon^E,\tau,\Omega) = \frac{1}{2}\varepsilon^E : D(\Omega) : \varepsilon^E + g\Omega : \varepsilon - \frac{1}{2\tau_0} C(\Omega)\tau^2 - \tau K(\Omega) : \varepsilon^E$$

Conjugation relationship

Stress

$$\sigma = \frac{\partial \Psi_S(\varepsilon^E,\tau,\Omega)}{\partial \varepsilon^E} = D(\Omega) : \varepsilon^E + g\Omega - \tau K(\Omega)$$

Stress evolution

Damage-driving force

$$Y = -\frac{\partial \Psi_S(\varepsilon^E,\tau,\Omega)}{\partial \Omega} = Y_1 + Y_2$$

Y_1 | Y_2

$$Y_1 = -g\varepsilon - \alpha(tr\varepsilon)\varepsilon - 2\beta(\varepsilon \cdot \varepsilon)$$

$$Y_2 = \frac{1}{2\tau_0}\frac{\partial C(\Omega)}{\partial \Omega}\tau^2 + \tau \frac{\partial K(\Omega)}{\partial \Omega} : \varepsilon^E$$

${Y_{1a}}^+$ | ${Y_2}^d$

$${Y_{1a}}^+ = -g\varepsilon^+$$

$${Y_2}^d = A \cdot \tau \cdot \alpha_\tau(\alpha + 2\beta) tr(\varepsilon^{E+})$$

${Y_d}^+$

$${Y_d}^+ = -g\varepsilon^+ + A \cdot \tau \cdot \alpha_\tau(\alpha + 2\beta) tr(\varepsilon^{E+})$$

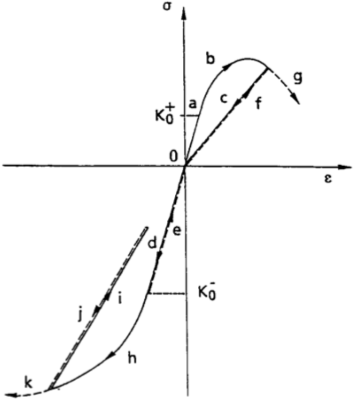
Damage criterion

$$f_d({Y_d}^+,\Omega) = \sqrt{\frac{1}{2}{Y_d}^+ : {Y_d}^+} - (C_0 + C_1\Omega)$$

[http://www.visualphotos.com]

[Zhu & Arson, *Acta Geotechnica*, 2013]

Unilateral Effects of Crack Closure



Chaboche, 1993

- Continuum Damage Mechanics : unilateral effects

  tensor damage variable Ω obeying an evolution criterion (f_d) similar to plasticity

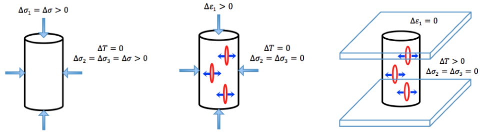
  $\Rightarrow$ Kuhn-Tucker consistency equations, damage cannot decrease

  $\Rightarrow$ neutralization of crack-induced damage in compression only (closed cracks)

- Unilateral recovery of stiffness

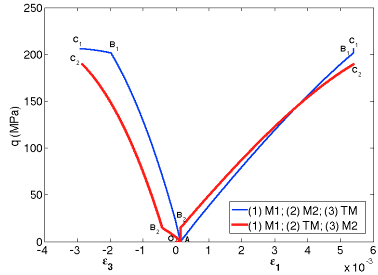
$$\mathbf{C_{eff}}(\mathbf{D}) = \boldsymbol{C}(\boldsymbol{D}) + \eta \sum_{k=1}^{3} H(-tr(\mathbf{P_k} : \varepsilon))\mathbf{P_k} : (\mathbf{C_0} - \mathbf{C}(\mathbf{D})) : \mathbf{P_k}$$

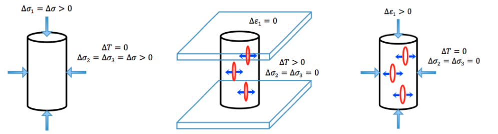
Stress-Strain Relation and Damage Evolution with Temperature



$O \rightarrow A$: **isotropic confinement (M1)** $A \rightarrow B_1$: **triaxial compression (M2)** $B_1 \rightarrow C_1$: **confined heating (TM)**

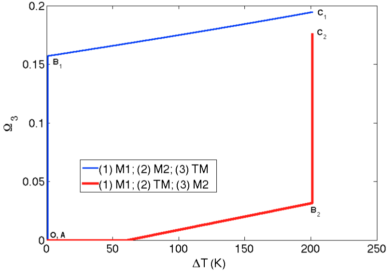
Sequence 1



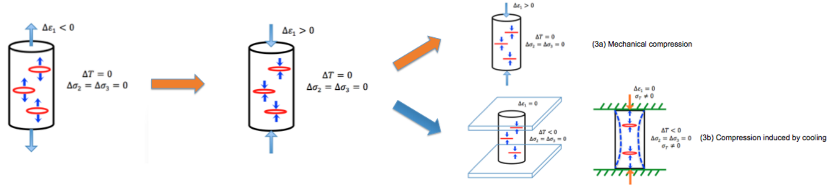


$O \rightarrow A$: **isotropic confinement (M1)** $A \rightarrow B_2$: **confined heating (TM)** $B_2 \rightarrow C_2$: **triaxial compression (M2)**

Sequence 2



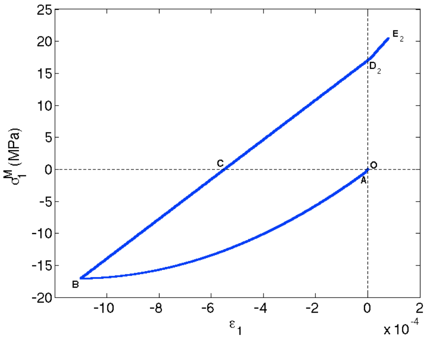
Comparison of TM to Mechanical Crack Closure

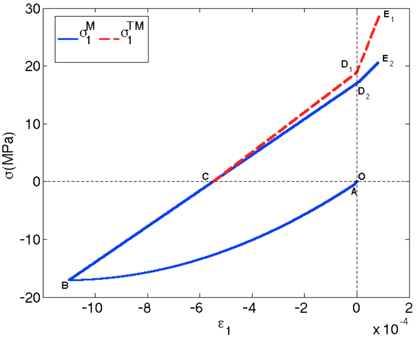


(1) Uniaxial tension
OA (elastic) – AB (damage)

(2) Compression: release of tensile stress
BC (“elastic” crack closure)

(3) Further compression: full closure and unilateral effects
CD (closure of residual cracks) – DE (unilateral effects)





CDM Framework of Healing

"Net Damage" :

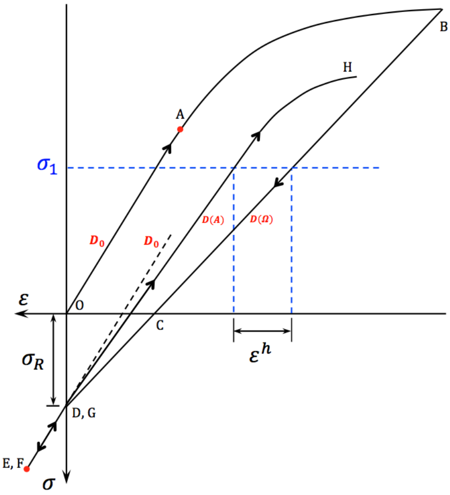
$$\boldsymbol{A} = \boldsymbol{\Omega} - \frac{h}{3}\,\boldsymbol{\delta}$$

h/V_{REV} work-conjugate to g_l^c

Diffusion Equation for Healing :

$$u = U_0 - h, \qquad \frac{\partial u}{\partial t} = D_c \nabla^2 u$$

$$U_0 = tr(\boldsymbol{\Omega})_{t=0}$$



[Arson, Xu, Chester, *Geotech. Letters*, 2012]

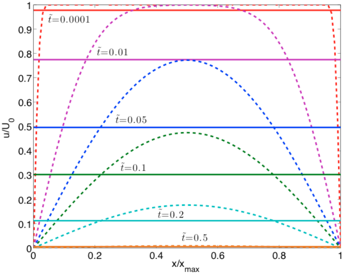
Diffusion Equation of Healing Evolution

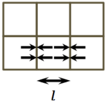
$$\frac{\partial u}{\partial t} = D_c \nabla^2 u$$
$$u(x=0,t)=0, \quad u(x=x_{max},t)=0$$
$$u(x,t=0)=U_0$$

$x_{max}/2 = l/2$: grain radius

$$u(x,t) = \frac{4U_0}{\pi} \sum_{i=1,3,5,\ldots}^{\infty} \frac{e^{-\lambda_n^2 D_c t} sin(\lambda_n x)}{n}$$

$$\lambda_n = \frac{n\pi}{x_{max}}$$

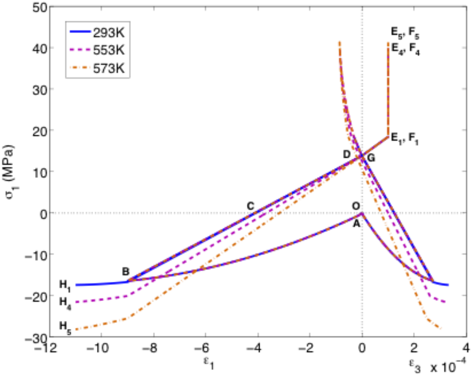




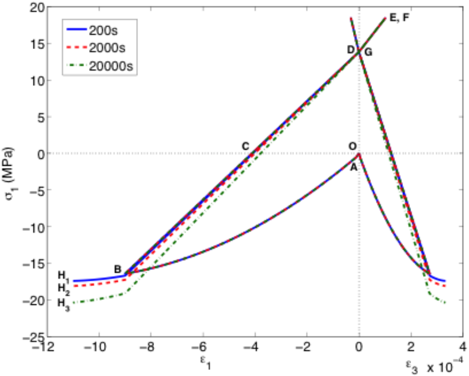
$$u(t) =< u(x,t) >= \frac{1}{x_{max}} \int_0^{x_{max}} u(x,t)dx = \frac{8U_0}{\pi x_{max}} \sum_{i=1,3,5\ldots}^{\infty} \frac{e^{-\lambda_n^2 D_c t}}{n\lambda_n}$$

Simulation of stress paths at the material point

Tension-Relaxation-Recompression-Creep-Reloading



Healing period of 200s, different temperatures.



Room temperature, various healing periods.

Goal : Finding Texture Descriptors to Match Phenomenological Scales

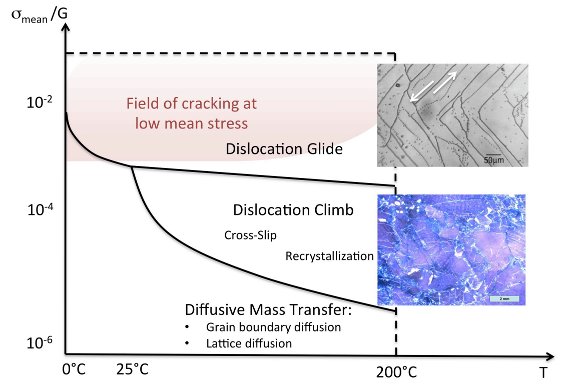


Figure 1: Micro-mechanism map for halite at repository conditions (dashed box).

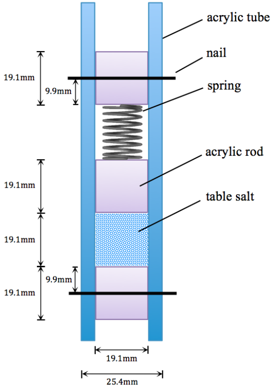
Diagram and data from (Hunsche, 1981; Senseny et al., 1992; Fam et al., 1998). Photos from (Schleder & Urai, 2005; Barber et al., 2010).

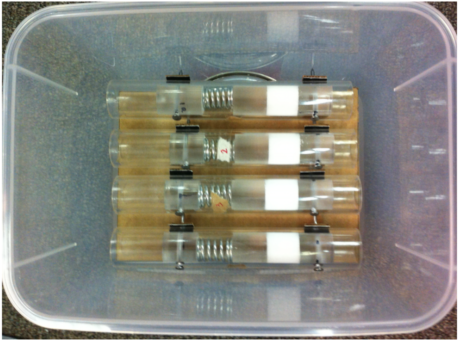
T temperature
σ_{mean} mean stress
$\boldsymbol{\sigma_d}$ deviatoric stress
G rock shear modulus
R=8.3142.10^{-3} kJ/K.mol
K Boltzman constant
Q activation energy
D diffusion coefficient
d grain diameter
Ω microstructure volume

Dislocation Glide	$\dot{\varepsilon} = A\exp\left(-\frac{B\sigma_d}{RT}\right)\exp\left(-\frac{Q}{RT}\right)$
Dislocation Climb	$\dot{\varepsilon} = A\times\left(\frac{\sigma_d}{G}\right)^n\exp\left(-\frac{Q}{RT}\right)$
Diffusion	$\dot{\varepsilon} \propto K\times\left(\frac{D}{d^2}\right)\times\left(\frac{\Omega\sigma_d}{kT}\right)$

Macroscopic Creep Laws

Creep tests in table salt

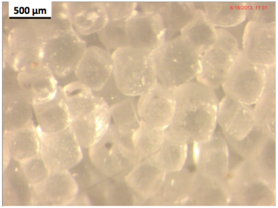




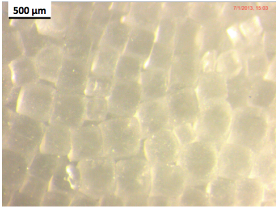
Experimental set-up for the creep tests performed on table salt

[Zhu & Arson, *Geotech. & Geol. Eng.*, under review]

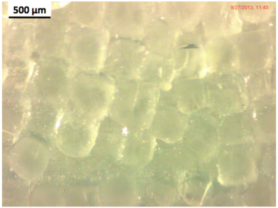
Microscopic observation of creep processes in table salt



(a) 18 days

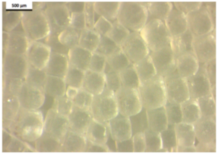


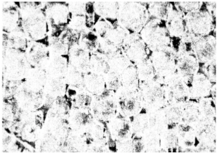
(b) 31 days

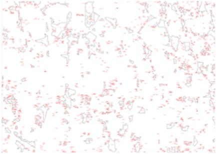
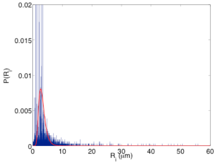


(c) 120 days

Statistical image analysis





Relationship between fabric descriptors and phenomenological variables ?

- **Distribution of sizes** : area of voids intercepted in the section : A_v
- **Distribution of orientations** : radius of the void space projected in a plane of normal j : R_j, $j = 1, 2, 3$

Micro-macro relationships

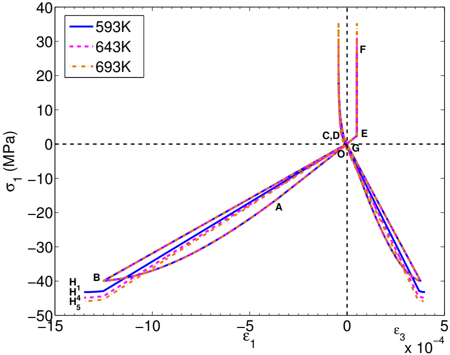
$$\Delta\varepsilon \rightarrow \Delta\boldsymbol{A} \rightarrow \overline{R}_1,\, \overline{R}_2,\, \overline{R}_3 \quad \underbrace{\rightarrow}_{\overline{R}_j = \int R_j p_j(R_j) dR_j} \quad p_j(R_j)$$

$$\Delta\varepsilon \rightarrow \Delta n_{3D} \rightarrow \Delta n_{2D} \quad \underbrace{\rightarrow}_{\overline{n}_{2D} = \int \frac{A_v}{A_{REV}} p_A(A_v) dA} \quad p_A(A_v) = a \times A_v^t$$

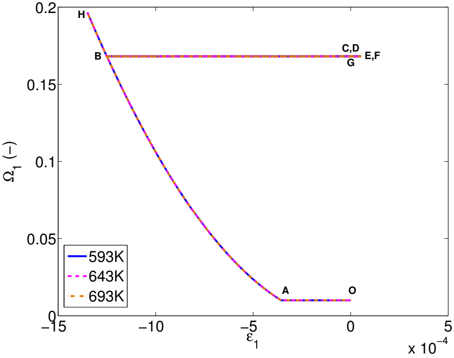
$$p_A(A_v),\, p_j(R_j) \rightarrow \sigma_R^{(i)} \rightarrow \boldsymbol{\sigma} \rightarrow \varepsilon^{el},\, \varepsilon^{ed},\, \varepsilon^{id}$$

[Zhu & Arson, *Geotech. & Geol. Eng.*, under review]

Effect of temperature : macroscopic variables

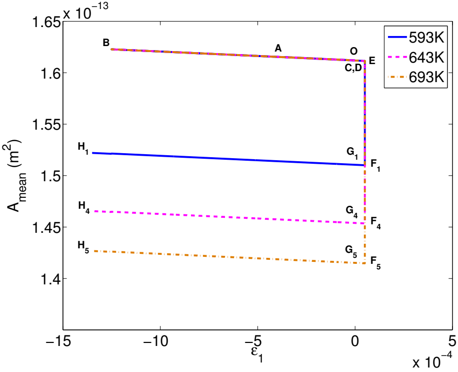


Stress/strain plot (controlled in deformation)

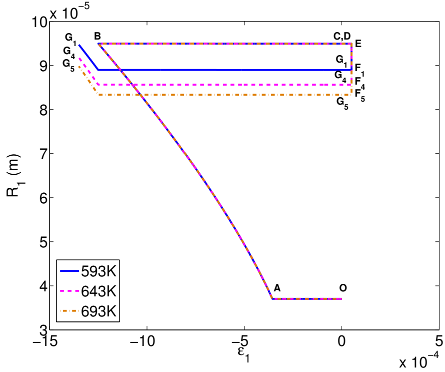


Net damage in dir. 1 (horizontal crack planes)

Effect of temperature : microscopic descritpors



Area of voids intercepted in the section : A_v

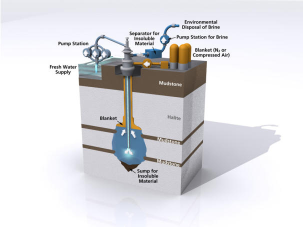


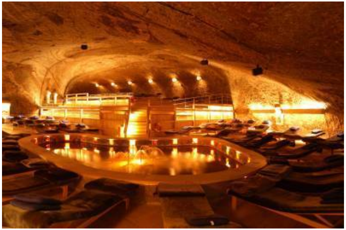
Radius of the void space projected in the plane of normal to dir. 1 : R_1

Energy and waste storage in salt rock

Radioactive Waste Disposals (Asse Salt Mine, Germany)

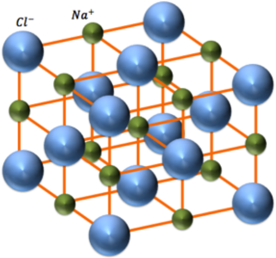
Production vs. demand : **CAES (e.g., wind turbines)** & Natural Gas Storage





[Tunneltalk.com, Word Press]

Microscopic model of viscoplasticty



FCC Crystal of NaCl

l^{th} sliding system of the mono-crystal :

$$a^l_{ij} = \frac{n_i m_j + n_j m_i}{2}$$

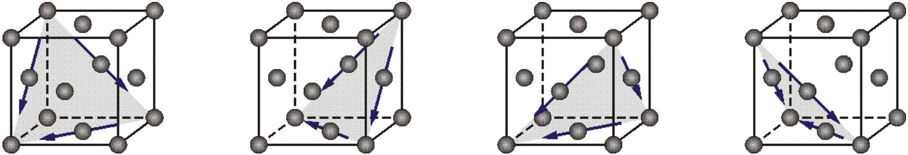
n is the vector normal to the sliding plane
m is the unit sliding vector

Mono-crystal viscoplastic deformation :

$$\dot{\epsilon}^{vp}_{ij} = \sum_{l=1}^{L} \dot{\gamma}^l a^l_{ij}, \quad \tau^l = \boldsymbol{\sigma} : \mathbf{a}^l$$

$L = 6$ (2 independent mechanisms)

www.kochmann.caltech.edu

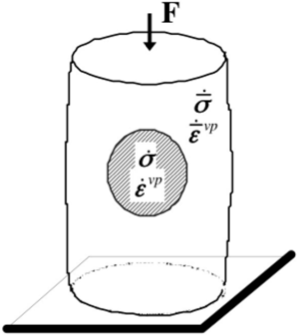


Self-consistent homogenization

Viscoelastic **self-consistent** model of Weng (1982) :

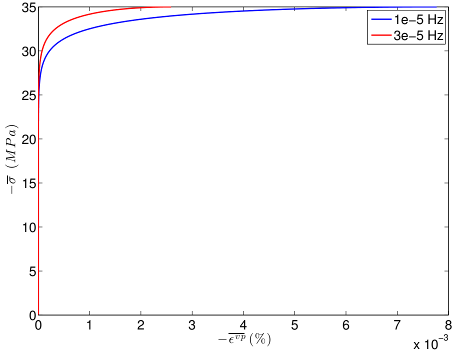
$$\dot{\boldsymbol{\sigma}} = \dot{\bar{\boldsymbol{\sigma}}} + 2\mu(1-\beta)(\dot{\bar{\epsilon}}^{vp} - \dot{\epsilon}^{vp})$$

$$\beta = \frac{2(4-5\nu)}{15(1-\nu)}$$

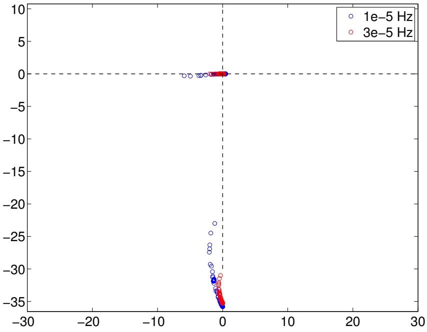


Matrix-inclusion model

Monotonic loading : influence of loading rate



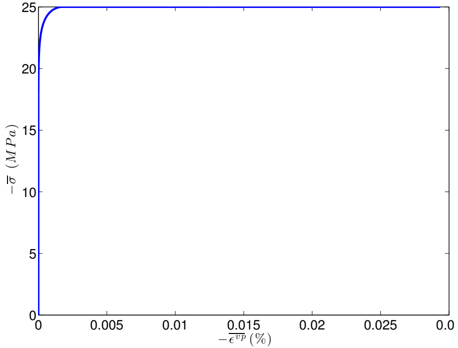
Macro. stress vs. macro. viscoplastic strain



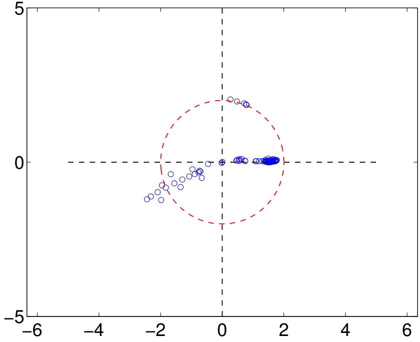
Major microstresses in axial and lateral dir.

[Zhu, Pouya & Arson, *ARMA Symposium*, 2014]

Creep test : initiation of damage



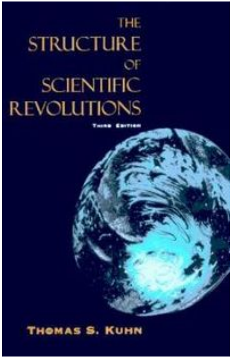
Macro. stress vs. macro. viscoplastic strain



Major microstresses in axial and lateral dir.

[Zhu, Pouya & Arson, *ARMA Symposium*, 2014]

Models shall change with paradigms...



"While learning a new paradigm, the scientist gets at the same time a theory, methods and judging criteria - usually, in the form of an intricate mixture. That is the reason why, during paradigm shifts, there is generally a significant change in the criteria which determine the legitimacy of problems raised and solutions proposed."

T.S. Kuhn, The Structure of Scientific Revolutions, Chap. VIII : Nature and Necessity of Scientific Revolutions, 1962.


Let us be realistic. Models are just models...